

FINITE-DIFFERENCE TIME-DOMAIN ANALYSIS OF MICROWAVE FERRITE DEVICES

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Abstract

Finite-Difference Time-Domain Method is extended to gyromagnetic media using a time domain approach by solving Maxwell's curl equations and equation of motion of the magnetization vector in consistency. New update equations are derived to include finite size ferrite material magnetized in a general direction. New code includes non-uniform magnetization due to demagnetization effects as well. In order to validate our formulation, a full wave analysis of a thin film isolator is performed in 3D. Numerical results are then compared with measurements and a good agreement has been achieved. The method presented in this paper can be applied for other microwave ferrite devices such as circulators and phase shifters.

1 Introduction

The electromagnetic wave propagation on microstrip lines having magnetic and ferrimagnetic substrates was studied by Pucel [1] [2]. Wen [3] introduced coplanar waveguide (CPW) in 1969 and experimentally investigated the differential phase shift of a CPW on a yttrium iron garnet (YIG) substrate. He also measured the isolation and the insertion loss of a resonant isolator based on CPW structure. El-Sharawy [4] performed a full wave analysis of CPW and slot lines on magnetic substrates using integral equation technique with transmission matrix method.

Currently, frequency domain full wave techniques, such as the Method of Moments (MoM) and the Finite Element Method (FEM) dominated the research in the area of numerical modeling of microwave ferrite devices. There has been little work in this area using a time domain approach. Time domain approach is computationally efficient in the analysis of wide-band devices such as circulators and isolators since it is possible to obtain wide band frequency results by fre-

quency transforming the time domain data. Luebbers [5] extended Finite-Difference Time-Domain (FDTD) method to isotropic and electrically dispersive media using a convolution process to evaluate frequency dependent constitutive parameters. Hunsberger [6] extended FDTD for magnetized plasmas and used frequency domain permittivity tensor and recursive convolution. Pereda [7] used a time domain approach solving Maxwell's curl equations and the equation of motion of the magnetization vector. They applied their technique to the calculation of propagation constants and dispersion characteristics [8] of transversely magnetized ferrite loaded waveguides, both in 2D. However, their technique requires linear interpolation to calculate some magnetic field components which can increase the global error specially in the analysis of slow wave devices.

This paper presents a new technique to extend FDTD to include magnetized ferrites. FDTD method was introduced for the electromagnetic field analysis problems by K. S. Yee in 1966 with the application of Yee's algorithm [9]. Gyromagnetic media is modeled in time domain by solving Maxwell's curl equations and equation of motion of the magnetization vector in consistency. Magnetic losses are included in the formulation by means of ferromagnetic resonance linewidth ΔH . New FDTD update equations are derived for a ferrite biased in a general direction which require no interpolation. New code includes non-uniform magnetization and finite size of the magnetic materials. The technique presented here and in [12] is applied to a thin film isolator (TFI) structure and a good agreement has been achieved with the measurements.

2 Theory of Magnetized Ferrite

The well-known magnetic properties of ferrites are due to the existence of magnetic dipole moments that are mainly caused by electron spin [13]. A sufficient

magnetic dc bias can cause magnetic dipoles to align in the direction of bias field and precess around it. This phenomenon is widely used to construct various microwave devices such as isolators, circulators and phase shifters. However, the introduction of ferrimagnetic materials to microwave applications is due to their high resistivity at microwave frequencies. This property of ferrimagnetic materials reduces high eddy current losses which are inversely proportional to their resistivity [15]. Maxwell's curl equations in differential form for homogeneous, linear, anisotropic, source free and lossy medium are

$$\begin{aligned}\nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{H} &= \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t}\end{aligned}\quad (1)$$

Constitutive relations between \vec{B} and \vec{H} , and \vec{D} and \vec{E} are as follows

$$\begin{aligned}\vec{B} &= \bar{\bar{\mu}} \vec{H} \\ \vec{D} &= \bar{\bar{\epsilon}} \vec{E}\end{aligned}\quad (2)$$

Here, $\bar{\bar{\mu}}$ is the permeability tensor and $\bar{\bar{\epsilon}}$ is the permittivity tensor. Since ferrites are electrically isotropic materials, the permittivity tensor reduces to a scalar ϵ . The permeability tensor is a 3×3 matrix whose elements are derived from the differential equation of motion of the magnetization vector. Equation of motion of the magnetization vector for a unit volume ferrite, including the phenomenological loss term α , can be written as

$$\frac{\partial \vec{M}}{\partial t} = -\mu_0 \gamma \vec{M} \times \vec{H} - \frac{\alpha}{|M|} \left(\vec{M} \times \frac{\partial \vec{M}}{\partial t} \right) \quad (3)$$

where \vec{M} is the magnetization vector, γ gyromagnetic ratio, and \vec{H} the total magnetic field intensity vector internal to ferrite. The second term in the right hand side of (3) is known as Gilbert's damping term [16].

Assuming that a sufficiently large magnetic dc bias field H_0 in x direction associated with a small ac magnetic field \vec{H} perpendicular to bias direction is applied to an infinite ferrite medium. The ferrite will be saturated with a saturation magnetization of M_s in x direction and due to the ac magnetic field, ac magnetization \vec{M} will be generated. Total magnetic field \vec{H}_T and magnetization \vec{M}_T can now be written

$$\begin{aligned}\vec{H}_T &= \hat{a}_x H_0 + \vec{H} \\ \vec{M}_T &= \hat{a}_x M_s + \vec{M}\end{aligned}\quad (4)$$

and equation of motion of the magnetization vector becomes

$$\frac{\partial \vec{M}_T}{\partial t} = -\mu_0 \gamma \vec{M}_T \times \vec{H}_T - \frac{\alpha}{|M_T|} \left(\vec{M}_T \times \frac{\partial \vec{M}_T}{\partial t} \right) \quad (5)$$

Following approximations are made

$$\begin{aligned}H_0 &\gg |H| \\ M_s &\gg |M| \\ |M_T| &\approx M_s\end{aligned}$$

Substituting (4) in (5) using above approximations and applying central finite-differences, we obtained the FDTD update equations for H_y and H_z field components for an infinite ferrite medium biased in x direction.

3 Numerical Results and Experimental Verification

Fig. 1 and Fig. 2 shows cross-sectional and top views of the isolator. The strips on the YIG substrate are made from gold. YIG substrate has a thickness of 47 microns and stays on top of a 400 microns thick dielectric substrate backed by a ground plane. Structure is shielded by a metallic wall on top, and side walls. The strip in the middle serves as a ground conductor and is 250 microns wide. Neighboring strips have widths of 40 microns and the gap between the strips is 30 microns. Ground strip is shorted as shown in Fig. 2. Each short is 40 microns long. Gold strips are 1 microns thick and the structure is 2000 microns in length. Dielectric and YIG substrates have relative permittivities of $\epsilon_r = 10.0$ and $\epsilon_r = 15.0$, respectively. Dielectric substrate is assumed to have a loss tangent of $\tan \delta = 0.002$ and electrical loss of YIG is included as $\tan \delta = 0.0002$. Magnetic losses of thin YIG substrate are introduced in the form $\alpha = 0.02$. H_0 is taken to be 1500 Oe in x direction and $M_s = 1780$ Gauss. In FDTD calculations, non-uniform mesh is used in z direction with $\Delta z = 2$ microns cells in the vicinity of YIG-conductor interface and away from the interface

Δz expands in size. Cell sizes in x and y directions are $\Delta x = 40$ and $\Delta y = 10$ microns, respectively.

The TFI's operating mechanism depends on magnetostatic waves (MSW) propagation in the structure. MSW can be grouped into two main categories; magnetostatic surface waves (MSSW) and magnetostatic volume waves (MSVW). Only, MSSW is of our interest, since the nonreciprocal behaviour of TFI is generated by MSSW in this specific application. The structure is excited by a Gaussian pulse voltage at port 1. Mur 1st order absorbing boundary conditions (ABC) [17] are used in front and back planes of the structure. Mur ABC is derived from the one way wave equation which allows numerical wave to propagate only in one direction. For insertion loss calculations, dc bias field H_0 and saturation magnetization M_s should be both negative quantities. For isolation calculation, H_0 and M_s should be positive. This is due to the MSSW propagation in the structure. Basically, during $+x$ bias, MSSW will stick to highly dispersive YIG-dielectric interface and input signal at port 1 will be attenuated. Meanwhile for $-x$ bias, MSSW will stick to YIG-conductor interface.

In FDTD calculations, dielectric substrate thickness is truncated to 96 microns due to computer resources and run time limitations. Air region above the strips was extended to 72 microns. Fig. 3 shows comparison between the measurement and FDTD calculation of insertion loss. Good agreement has been achieved except FDTD result has narrower bandwidth than the measurement. Insertion loss is observed to be -6 dB at 7.5 GHz. Isolation calculation is in good agreement with the measurement in terms of magnitude and frequency as shown in Fig. 4. However a discrepancy is observed below 7.1 GHz. This might be due to the edge effects which are not included in the computation. Edge effects alter bandwidth significantly [18] and are under investigation currently.

4 Conclusions

A new technique is presented to include magnetized ferrites with the FDTD algorithm. Gyromagnetic media is modeled in time domain by solving Maxwell's curl equations and equation of motion of the magnetization vector. New FDTD update equations are derived for a ferrite biased in x direction and the technique presented here is applied to a slow wave device namely thin film isolator using a 3D code. Numerical results are compared with the measurements and good agreement in insertion loss calculation and fair agree-

ment in isolation calculation have been achieved. This technique can be applied for other dc bias directions as well as other microwave ferrite devices.

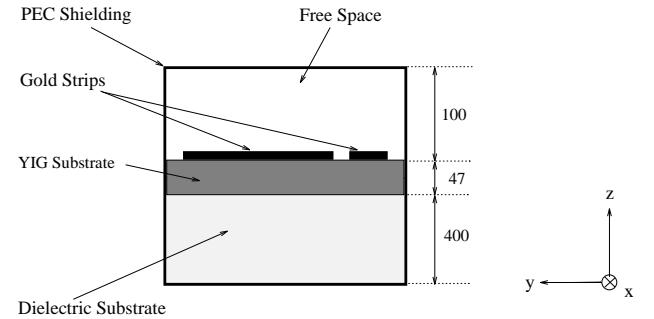


Figure 1: Cross sectional view of the thin film isolator, all dimensions are in μm .

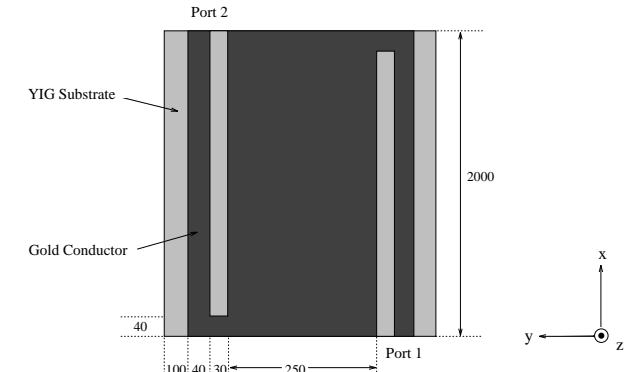


Figure 2: Top view of the thin film isolator, all dimensions are in μm .

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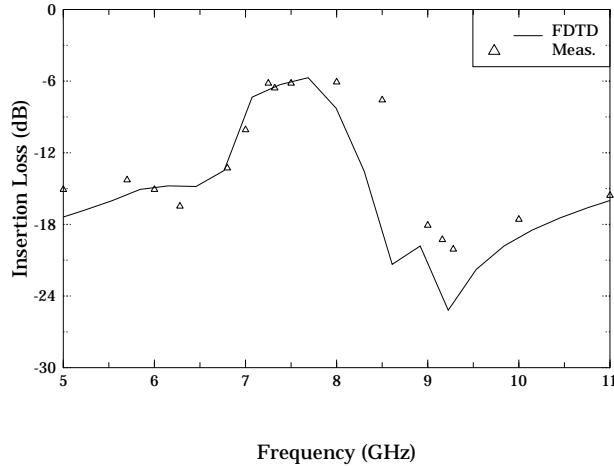


Figure 3: Comparison of FDTD calculation and measurement of insertion loss.

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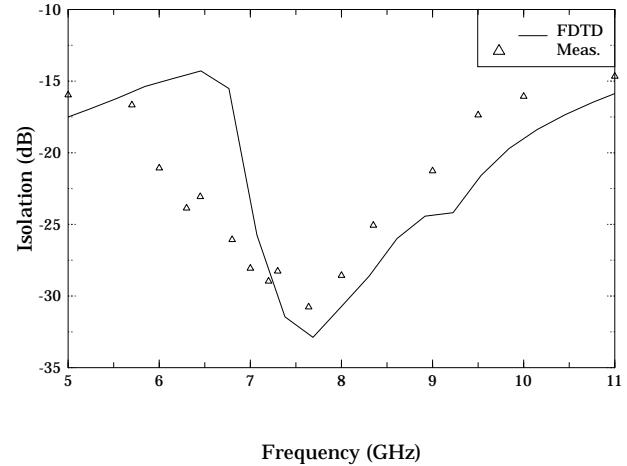


Figure 4: Comparison of FDTD calculation and measurement of isolation.

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